Towards Real-time Safe Optimization for Autonomous Vehicles under Uncertainties

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Enforcing safety while preventing overly conservative behaviors is essential for autonomous vehicles (AVs) to achieve high task performance [1]. To achieve this goal, AVs must generate safe, feasible, and comfortable trajectories during real-time replanning iterations [2], [3]. Despite significant advancements, addressing potential safety hazards remains a critical challenge under uncertainties (e.g., varying terrain and sensor occlusions), particularly for safety-critical high-speed autonomous driving [4], [5]. Additionally, these uncertainties may lead to abrupt maneuvers (e.g., sudden lane changes or decelerations) to address unforeseen contingencies, thus disrupting driving consistency and compromising task efficiency [6], [7]. Moreover, real-time replanning in such environments involves addressing intricate constraints and objectives simultaneously for multi-objective optimization tradeoffs [8]. These challenges are further exacerbated in dense and interaction-heavy traffic environments, making real-time optimization computationally intensive [9], [10]. In light of those real-world problems, my research vision is to answer: *How to safely deploy AVs that avoid over-conservative behaviors and maintain driving consistency in real time under uncertainties?*

One key underlying factor to the safety concern stems from uncertainties in both internal system models (e.g., unknown parameters) and external environments (e.g., unknown intentions of dynamic obstacles) [11], [12]. These uncertainties become critical in contingency scenarios, where potential risks cannot be predicted with certainty [13], [14]. My past research addresses this problem from two perspectives: 1) Developing consensus spatiotemporal safety barrier within a scenario tree structure to address potential contingencies when historical obstacle data is unavailable (e.g., occluded phantom vehicles). 2) Anticipating the influence of uncertainties on the system state through fast online Bayesian learning, and leveraging control theory to design asymptotically stable safety barrier certificates. Furthermore, my work enables fast optimization while balancing multiple constraints, such as safety and driving stability, in high-dimensional, nonlinear planning and control problems under dense obstacle environments. To achieve this goal, we explore constraint transcription and decompose the nonlinear optimization problem using parallel optimization methods, including multithreading techniques, multiple shooting [15], [16], and the alternating direction method of multipliers (ADMM) [17]. These approaches streamline the optimization process while ensuring compliance with safety constraints, as validated through both theoretical analysis and real-world experiments.

A. *Safety under Uncertainties*

Ensuring safety in AVs requires confining the state of the system to a provably safe subset of the state space. Traditional methods enforce safety via explicit constraints [12], [18], [19], [20]. However, they typically neglect unmodeled epistemic uncertainty (e.g., sensor occlusion) and uncompensated aleatory uncertainty (e.g., terrain deformation) in contingency scenarios [11]. My past work addresses these limitations through two aspects: 1) **Consensus safety**: Tackling potential risks arising from epistemic uncertainty (e.g., sensor occlusion, unanticipated dynamic obstacles). 2) **Safety recovery**: Enabling AVs to recover from unsafe states (excluding collisions) to a safe state under aleatory uncertainty (e.g., stochastic terrain parameter variation). For instance, the ego vehicle (EV) can asymptotically restore a safe following distance after a sudden cut-in by surrounding vehicles (SVs).

To anticipate the evolving states of the EV and SVs, we develop a bi-convex spatiotemporal safety barrier [9]. This module employs an adaptive barrier coefficient strategy across the optimization horizon to account for the prediction inaccuracy of the future trajectories of SVs. By progressively enlarging the solution space in later planning phases, the strategy reduces conservatism while facilitating stable adjustments, thus balancing safety and task performance. To further tackle the environment perception uncertainties, such as sensor occlusion, we introduce a consensus spatiotemporal safety barrier within a scenario-tree structure [21], [22]. The reachability analysis is employed to dynamically assess potential risk configurations for local trajectories during safety barrier design. This strategy ensures that all generated trajectories share a common consensus segment, guaranteeing persistent safety within the trajectory space despite perception uncertainties. Moreover, the biconvex structure of spatiotemporal safety barrier constraint enables decomposition into low-dimensional convex formulations, facilitating fast optimization while rigorously preserving safety guarantees.

For unexpected safety violations, my previous research proposes a stochastic stabilizing control barrier framework [23]. By integrating incremental Bayesian learning, we efficiently update the kernel matrix and its inverse in the

Gaussian processes (GPs) using Woodbury matrix identity optimizations. Consequently, the learning complexity of GPs is reduced from $O(n^3)$ to $O(n^2)$ [23], [24]. This method embeds the learned uncertainty bounds into control barrier constraints, ensuring that the EV can safely converge to a safe state in the presence of varying model uncertainties. For instance, the EV can recover a safe following distance after abrupt disturbances, acting as a real-time safety recovery filter under model uncertainties.

B. Safe and Consensus Parallel Optimization

Safe and efficient trajectory optimization is critical for AVs to safely execute tasks while incorporating real-time sensor feedback in dynamic environments. A critical challenge stems from the non-holonomic kinematic constraints inherent to AVs, which, when coupled with safety-critical boundary conditions, result in nonlinear and non-convex optimization problems [3]. In obstacle-dense environments, these issues become more pronounced, as conventional sequential solvers struggle to achieve real-time performance. Furthermore, achieving Pareto-optimality in balancing multiple objectives, such as safety and comfort, poses a significant challenge due to the inherent tradeoffs between these criteria [8].

Our prior work addresses these issues through a spatiotemporal receding horizon control (ST-RHC) framework, which integrates planning and control through a multiple shooting formulation [25]. To mitigate local optimal issues in dense traffic scenarios, we extend the ST-RHC framework into a real-time parallel trajectory optimization strategy [26]. This strategy employs multithreading and multiple shooting techniques to blend discrete maneuver decisions into continuous parallel trajectory optimization, achieving replanning frequencies that exceed 50 Hz in dense traffic flow.

To further address Pareto-optimality, we introduce a barrier-enhanced homotopic optimization (BPHTO) method [9]. The proposed BPHTO exploits the bi-convexity of the kinematics of the EV and the spatiotemporal control barrier to formulate a bi-convex optimization problem, striking a balance between safety and task performance. Using reachability analysis, we devise a warm initialization goal sample strategy to determine discrete maneuver homotopy for BPHTO in a receding horizon planning manner. This allows the EV to respond adeptly to SVs with enhanced driving consistency. Additionally, we decompose the BPHTO into several low-dimensional Quadratic Programming (QP) subproblems via over-relaxed ADMM iterations [27], ensuring real-time feasibility.

Considering motion consistency under perception uncertainties, we propose a risk-aware consensus parallel optimization strategy [22] and a contingency planner [21]. These methods enable each trajectory to share a common consensus segment while addressing various risk scenarios, ensuring both safety and motion consistency in dense traffic. By utilizing discrete-time barrier function theory, we ensure trajectory safety through forward invariance within a consensus safe set. We further exploit the biconvex properties of the constraints and decompose them into a series of low-dimensional QP subproblems via consensus ADMM. This strategy ensures each generated feasible trajectory adheres to the same consensus segment while enabling large-scale optimization in real time. Extensive validation using real-world traffic datasets and hardware experiments demonstrate robust safety and motion consistency in occluded scenarios, with stable maximum computation times (< 100 ms) for problems involving up to 5175 variables and 7860 constraints.

For unstructured navigation tasks under epistemic uncertainty (e.g., sensor occlusion), we tailor the consensus ADMM approach into a branch model predictive control framework for safe robot navigation in occluded, obstacle-dense environments [28]. This method accelerates constraint evaluations via parallelized Jacobian computations and coordinates trajectory hypotheses. Real-world experiments on an Ackermann-steering robot validate its efficacy in obstacle-dense environments with occluded dynamic obstacles.

C. Future Research

Interaction-aware Safety Filter: Current safety filters typically neglect the bidirectional influence inherent in human-robot systems, where the action of ego agent influences human behavior [12], [29]. To address this gap, we propose to develop a real-time interaction-aware safety filter for AVs in crowded, partially observable environments (e.g., urban intersections). This initiative addresses three scientific challenges: (1) modeling reciprocal collision responsibility between ego agents and dynamic participants, (2) overcoming reactive behavior of existing safety filters (e.g., CBF-based strategies) with formal safety guarantees, and (3) achieving fast optimization in cluttered multi-agent scenarios (e.g., exceeding 100 Hz).

Learning Implicit Safety Constraints: A promising approach to capture latent safety rules in human-robot systems involves learning safety constraints from human demonstrations via model-based diffusion techniques [30]. We aim to train context-dependent safety preferences (e.g., yielding thresholds at unmarked crosswalks) while quantifying predictive uncertainty using probabilistic tubes. These tubes encode confidence bounds for learned behaviors, which are incorporated into a gradient-based safety filter via event-triggered thresholds. When learned policies approach these bounds, the safety filter intervenes to enforce worst-case safety guarantees by overriding data-driven policies.

Scalability in High-Dimensional Autonomous Systems: High-dimensional autonomous systems, such as humanoid robots, face challenges in balancing real-time safety and stability due to complex internal dynamics (e.g., upper-lower body coordination) [31]. To address this issue, we plan to use the data-driven Koopman operator [32] to embed nonlinear dynamics into a latent linear space for efficient optimization. Building on my prior consensus optimization work [22], [21], [28], we will extend these methods to coordinate subsystem interactions (e.g., arm-leg coordination in humanoids) using GPU-accelerated consensus ADMM iterations. This approach aims to handle mutual influences between subsystems while maintaining real-time performance.

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